

Running head: PASCAL'S TRIANGLE

Pascal's Triangle

The Many Tricks Held Within

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Abstract

Pascal's Triangle is a triangle in which numbers are arranged in rows to represent the coefficients of the binomial series. Although Blaise Pascal was one of the first to discover many of the interesting properties, he was not the first to discover the actual triangle. The first people to discover the triangle were the Chinese. Although this triangle is a lot of help when it comes to binomial expansion, it also has many more uses. It has been discovered that it may be use to work with probability. When working with combinations and permutations Pascal's Triangle comes to great use. Binomial theorem, combinations, and permutations each relate back to the triangle.

Background

Pascal's Triangle was first known in China as early as the 1200's. In 1261 the triangle appears to a depth of six in Yang Hui and then to a depth of eight in Zhu Shijie. In Yang Hui they attribute the triangle to Jia Xian, who lived in the eleventh century. They would use it to generate the binomial coefficients, as we do today. It wasn't until later that the method of solving quadratic and cubic equations was recorded. Around this time Jia Xian came about the square and cube root procedures to higher roots by using the array of numbers known today as Pascal's triangle. Years later after it first appeared in China, the triangle came to be known as Pascal's Triangle with Blaise Pascal. By making use of the already known array of binomial coefficients, he was able to develop many of the triangle's properties and applications within these writings (Smith, 1996).

Binomial Theorem

To construct Pascal's Triangle, begin with the number 1 at the tip which makes up the zeroth row. The first row (1 & 1) contains two 1's, both formed by adding the two numbers above them to the left and the right, in this case 1 and 0. Do the same to create the 2nd row: $0+1=1$; $1+1=2$; $1+0=1$. And continue on in this same pattern for each row. In this way, the rows of the triangle go on infinitely. A number in the triangle can also be found by using the formula:

$$(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$$

For example, in row 3, 1 is the zeroth element, 3 is the element number 1, the next is the 2nd element, and the last 1 is the 3rd element.

$$\begin{array}{ccccccc}
 & & & & 1 & & & & \\
 & & & 1 & & 1 & & & \\
 & & 1 & & 2 & & 1 & & \\
 & 1 & & 3 & & 3 & & 1 & \\
 & 1 & 4 & & 6 & & 4 & 1 & \\
 & 1 & 5 & 10 & & 10 & 5 & 1 & \\
 1 & 6 & 15 & 20 & & 15 & 6 & 1 & \\
 1 & 7 & 21 & 35 & & 35 & 21 & 7 & 1
 \end{array}$$

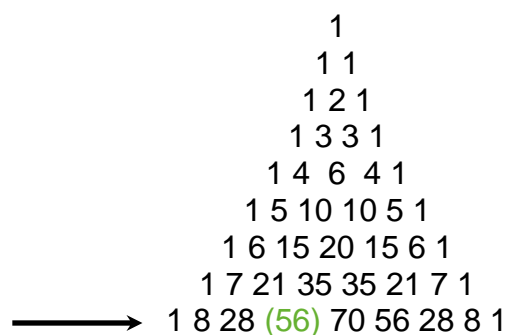
Although Pascal's Triangle is very interesting due to all of its patterns and properties, it also serves us in more practical ways. For example, it may be used as a tool to expand binomials. The numbers within any row of Pascal's Triangle coincide with the values of the coefficients of binomial expansions. Consider the equation $(x+y)^4$, after expanding it algebraically we end up with $1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$. Notice that

the coefficients of this binomial expansion, 1 4 6 4 1, match up to the numbers within row 4 of Pascal's Triangle.

Combinations and Permutations

The other highly important application of Pascal's Triangle is using it as a tool when working with combinations and permutations. Permutations are used when order does matter, and combinations are used when order does not matter. The Triangle

makes combinations easy as the numbers contained within can be used to find out how many subsets of elements can be formed from a set with unique elements. Say you want to calculate how many ways you can take 3 marbles out of a box with 8 marbles. Using the formula stated about, gives you 56. But looking at Pascal's Triangle, in the 8th row (remember that you start counting the first row as 0) and counting over to the 3rd position is 56. This is true for all numbers as long as r is smaller or equal to n . If not then the answer will equal zero (Dunham, 1990).



An example of a permutation is suppose we want to find the number of ways to arrange the three letters in the word CAT in different two-letter groups where CA is different from AC and there are no repeated letter. We can list them as: CA CT AC AT TC TA. So the permutation for this problem would be 6.

Probability

Probability is a way of expressing knowledge or belief that an event will occur or has occurred. For example, If a family wants to have 3 kids, what could be all the different outcomes? This problem can be looked at by using Pascal's Triangle.

B = Boy; G = Girl

1 child - B G

1 1

2 children - BB BG GB GG

1 2 1

3 children - BBB BGB BBG GBB BGG GBG GGB GGG

1 3 3 1

As shown above, the numbers of Pascal's Triangle are shown in each row. This pattern would continue on and on.

Conclusion

In conclusion, Pascal's Triangle has been a great discovery that is used for many things. Also, is used all over the world by many people. It can be used for binomial expansion, combinations, permutations, probability and many more things. But these are just a few things that have been discovered. There are still many more patterns that are waiting to be discovered inside Pascal's Triangle.

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