Pi: And Its Mathematical Contributions in the History of Aeronautics

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Abstract

As what could be one of the most mysterious and unknown figure in the mathematical realm, pi has without a doubt contributed to the evolvement many of the world's existing fields of necessity in the economy. One of these fields, is transportation, specifically the transportation of flight, in which this paper will identify. In this research paper, the history and necessities of pi will be discussed in relations to the history, and developments in aeronautical engineering. How pi has influenced the advancements of aeronautics will also be discussed as well as the differences in aeronautical engineering if an accurate enough value of pi did not exist. This essay will, in addition, study the how pi has developed and influenced life for mankind throughout history. The answers to how pi was calculated in ancient times and how its been used in aviation's advancements will be held in this research paper.

Probably even long since ancient civilizations have ever existed, and all throughout the ages, mankind has been fascinated by the idea of flight. In every culture around the globe there are thousands of myths, tales and stories of inanimate objects taking flight with the passenger of men. The history of aeronautics is, indeed, a long one, dating back to the times of the Greek Scientist, Aristotle (384-322 B.C.) who contributed with his reasonings of resistance of aerodynamic flows (Anderson, 1997, p.17). During the time when early compasses and straight edges were still in use for architectural constructions, Leonardo da Vinci made the first attempts at experimenting with aviation with his sketches of the ornithopter (Anderson, p.14).

Civilizations such, as the Egyptians and Babylonians, were successful in calculating a close value to the actual pi. Probably the oldest evidence of pi is the value of 3.16 in the Egyptian Rhind Papyrus (1650 B.C.) (Smith, 1996).But the earliest contributor to the advancement in flight, regarding pi, was the genius, Greek philosipher and mathematician, Archimedes (287-212 B.C.), who was the first to accurately approximate the value of pi, as well as devise many of the basic concepts in which the roots of aviation began. In his calculations, Archimedes discovered the closest estimated value of pi during his time. He did this by a method of exhaustion, in which he inscribed and circumscribed polygons inside a circle and outside a circle. Archimedes noticed that as the sides, of the polygons used, increased, the space between the circle's circumference and polygons' circumference *decreased*. This space would then get smaller and smaller as the polygon sides increase and approach the value of pi which is

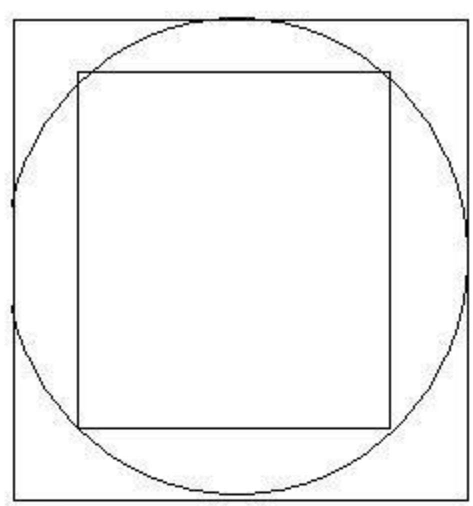


Figure 1

the ratio of the circle's circumference to its diameter (Archimedes Approximation, 1997). Archimedes method to finding his velue of pi is shown in the series of figures ones shown. In these examples, the square inscribing the circle has a larger area and possibility for pi. As can be seen, the hexagonal perimeters are much closer to the circle's circumference in comparison to the square's perimeters.

The five sided figures of a hexagon is used to be inscribed and circumscribed on a circle. The area between perimeters and circumference is now smaller and therefore is closer to the value of pi.

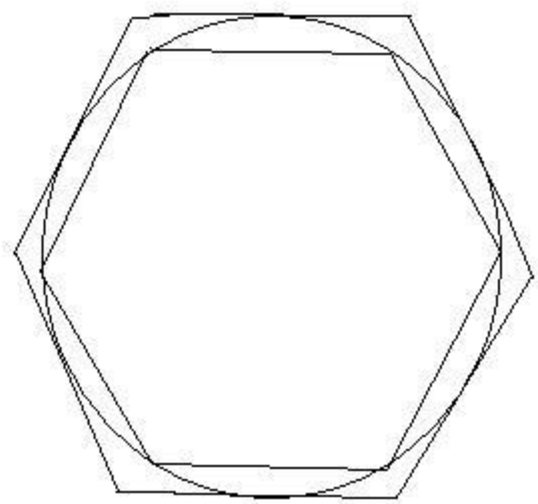


Figure1 : Method of Exhaustion

The second figure one below shows that π lies between the perimeters of the inscribed and circumscribed hexagon in the shaded area:

a<π>b

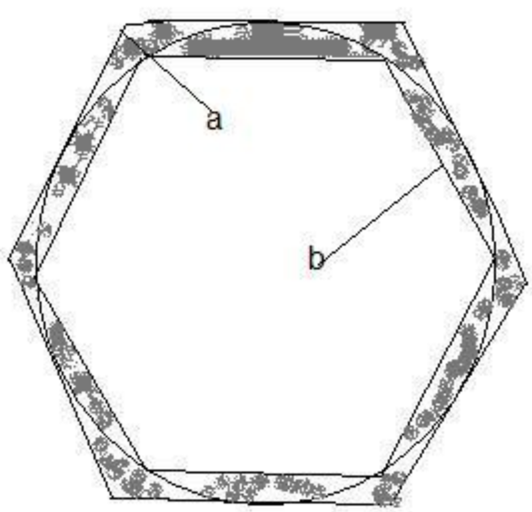


Figure1 : Method of Exhaustion

As the polygons' sides increase, the perimeters would slowly approach the circumference of the circle. Archimedes did this until he had the perimeters and circumference as close as possible; then by using a circle with a radius of ½ he was able to get π. From the method of exhaustion, Achimedes was able to approximate 3.1416 as π, a value that was much more proximate to the true pi value, when compared to other computed values of pi, such as 3.16 (Ancient Egyptians), 3.125 (Babylonians), and 3.162 (Early Greeks). Archimedes' π value was, in addition, more accurate when compared to other mathematicians, such as Ptomlemy-3.141666, Aryabhata-3.1416, and Ahmes-3.16 (Berlinghoff, 2002). Chinese mathematician Zu Chingzhi, however was able to compute a pi value of 3.14159292, more accurate than Archimedes, later on (Berlinghoff, 2002). Throughout history, there were many symbols and ways for representing pi, until finally the Greek symbol for the numerical digit of sixteen (π) was made the official global character for pi (Posamentier, 2004).

Interestingly enough, there is also a reverse method to Archimedes's method of exhaustion, done by a German thinker, Nicholas Cusanus(1401-1464) centuries later after Archimedes' death. Instead of using polygons to circumscribe and inscribe circles, Cusanus used circles to inscribed and circumscribed regular *n*-polygons and successfully generated the same results (Posamentier, 2004).

Evidence of π can be traced back to Biblical times, as the Bible verses 1 Kings 7:23-26 describes an approximate value of pi in the constructions of Solomon's palace (Smith, 1996). As much as π is used in architecture, it is no surprise that it would soon be purposeful in the construction of aeronautics also. This mysterious mathematical symbol has worked miracles and possibilities in the field of aeronautics because it constituters all the fundamentals of a circle. The circle is indeed a necessary geometrical figure needed for the existence of aviation and planes. For example, not only can circles and π be found in modern aviation but also in the oldest sketches of man dreamt flyers. As evident, Leonardo da Vinci did this by using the compass and straight edge to construct the sketches of his ornithopter, and thus increased and opened men's curiosity to explore further into the ideas of early aviation.

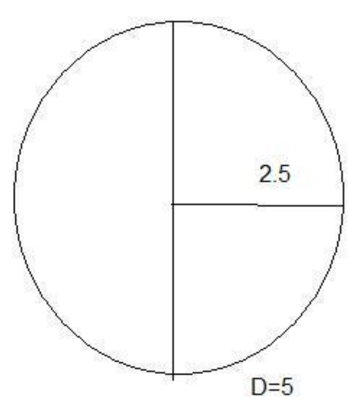
During the beginnings of the ideas and sprouting concepts of flight, the value of pi became increasingly accurate and known. Mathematicians began computing its decimal places and thus getting an increasing accuracy in the value of pi. This increasing accuracy established a precise number for early egineers to incorporate into their constructions of circles. Perhaps it was that Kitty Hawk was successful in 1903 because of the fact that in the 1900's John von Neumann was able to identify 2035 decimal places of π (Anderson, 1997). Engineering planes involve pi in constructing many of thei aspects such as propellers. For example if the tunnel space for a propeller had a diameter of 5 and to find the space that a propeller had to rotate in, the circumference of the circle needed to be known. In this situation, the simple but useful formula $C=2\pi r$ will need to be used. In this example the calculator value of π will be used.

$$D/2=r$$

$$C= 2(3.14...)(2.5)$$

$$C=15.707$$

Suppose this C value accounted for the exact maximum area the propeller would have. There will be a slight difference in the maximum propeller's space if the pi value of the Ancient Egyptians- 3.16, were used instead of the pi value used today.



The new circumference to this problem will then be $C=2\ (3.16)(2.5)$

$C=15.8$

If the unit for this problem was in meters, there would have been a 0.1 meter difference that could be enough to cause minor or even major errors for plane engineers.

One aeronautical theory, created by Ludwig Prandtl, during War World I began the first concepts and computations of airlift dynamics, which is how wings and other plane support units. His theory stated that the lift for an infinite wing is 2π per degree radian, a theory still in use today in the field of speed aeronautics (Anderson, 1997). Another equation useful to the world of aviation is one founded by Wilhem Kutta. Kutta's equation was specifically for the lift on a circular arc airfold at zero degree angle (Anderson, 1997). The equation was as follows:

$$L= 4\pi ap V^2 \sin^2(\theta/2)$$

The radius of the circular arc is r. And 2θ is the angle made by the arc at the circle's center.

Finding the ratios and measurements of flight wings and other airfoils are also another way in which pi can be used to assist in a solution.

In this way, when using angles and degrees to find and set locations for planes, pi has came in handy and grown in usefulness and mankind gradually discovers its mysteries. As the value of the decimal places of pi increases, the more precise coordinations and outsomes became over the years. And in modern day today, it has become accurate enough to keep the safety of our constructions and technologies that are not only those dealing with flight.

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