Powers of Two and Exponential Relationships: An Investigation into Exponential Partial Sums

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Abstract

This paper is concerned with the patterns that arise from simple series of numbers with exponents and their sums. I explored the relationships from one series to another and came upon a few interesting properties. If three numbers, $2^n+2^{n+1}+2^{n+2}$, are added together the sum can also be calculated by multiplying the first term 2^n by 7. This works with any three consecutive exponents. Similarly, any three numbers with exponents increasing by a common difference, *d*, with a common base, *b*, and an initial exponent, *n*, that is, $b^n+b^{n+d}+b^{n+2d}$, will always be equal to the initial term, b^n, times $b^{2d}+b^d+1$. The function $y = b^{2d}+b^d+1$ calculates the constant of multiplication for any series with a common exponential difference of *d* and a base of *b*. This results in the equation $b^n+b^{n+d}+b^{n+2d} = (b^{2d}+b^d+1)\times b^n$. There are also other generalizations and variations of this equation, one being to have an amount, *m*¸ that is multiplied to the difference, *d*, and added to the exponent, *n*. This would result in the series

$$b^n+b^{n+d}+b^{n+2d}+\ldots+b^{n+md} = (b^{(m)d}+b^{(m-1)d}+\ldots+b^{2d}+b^d+1)\times b^n$$

where there are a total of $m+1$ terms that are added together. I investigated these series and came upon these properties, though if I explored this topic I am certain more patterns would be clear. Nonetheless, the ones I have found are explained and illustrated in this report.