Hyperbolic Geometry: Shapes and Figures in a World without Euclid

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Abstract

This paper explores the possibility that a system of coordinate points, such as the one that exists in Euclidean geometry, can be created in non-Euclidean forms of geometry. Of the most well-known types of non-Euclidean geometry – elliptic and hyperbolic – this paper looks at solely hyperbolic geometry and the properties of the hyperbolic plane that may or may not allow for the creation of a basic coordinate system within the plane. The Poincaré disk model is used to view figures created in the hyperbolic plane on a two-dimensional surface that can be related to Euclidean geometry. The use of this model allows for the creation of an array of intersections between equally spaced vertical and horizontal lines that could potentially be used to mark coordinate points. However, certain characteristics of lines in the hyperbolic plane result in limitations to a coordinate system created in this manner.